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on Ship Stability

by

Prof. Dr. M. A. Shama

- 1- Shama, M. A., (UK-1968) "A Method for Calculating Ship Stability Curves", Shipbuilding and Shipping Record, Aug.
- 2- Shama, M. A., (UK-1969) "A Computer Program for Ship Stability Curves", Shipbuilding and Shipping Record, May.
- 3- Shama, M. A., (UK-1975) "The Risk of Losing Stability", Shipping World and Ship, Oct.
- 4- Shama, M. A., (Germany-1976) "On the Probability of Ship Capsizing", Schiff und Hafen, Sept.
- 5- Shama, M. A., (Egypt-1989) "Safety Requirements for Nile Tourist Vessels", Seminar on Future of Nile Tourism in Egypt, (In Arabic), Alex., Eng. Journal, Vol.28, No.2, April.
- 6- Shama, M. A., (Egypt-1993) "Ship Stability Assessment, Criteria & Risk", AEJ, July.
- 7- Shama, M. A., and others, (Egypt-2001), "Intact Stability of SWATH Ships", AEJ, Vol. 40

# THE RISK OF LOSING STABILITY

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In this paper by M. A. Shama, Associate Professor, Naval Architecture Department, Faculty of Engineering, Alexandria University, Egypt, the deficiency of initial stability as a general measure of ship stability is indicated. The variability of the main parameters affecting initial and dynamical stability is discussed. Particular emphasis being placed on the calculation of the risk of losing ship stability. Because of the lack of adequate statistical data to establish the mathematical model representing the random variation of the relevant parameters associated with initial and dynamical stability, a truncated normal probability density function is assumed. By virtue of statistical independence, initial stability and the reserve of dynamical stability also follow a truncated normal probability density function. However, for the sake of simplicity, the calculation of the risk of capsizing, or losing GM, is based on a normal probability density function.

The paper concludes (1) Deficient initial stability not only affects statical stability, but it also has an adverse effect on dynamical stability. (2) The variability of dynamical stability, has a marked influence on the risk of capsizing. (3) The risk of capsizing should be estimated and associated with the required minimum value of the reserve of dynamical stability.

properties of a ship. At small angles of inclination, the stability of displacement vessels is measured by the metacentric height, GM, and at large angles of inclination, the statical stability curve is normally used.

The magnitude of GM depends on loading condition, ie, cargo distribution, form and geometry of vessel, sea condition and speed of vessel. On a wave crest and in a following sea, GM may be seriously affected. The carriage of deck cargoes also has an adverse effect on GM. Consequently, GM should be treated as a random variable since the main parameters affecting its magnitude are not deterministic quantities. In this case, the assigned minimum value of GM should be associated with the probability of losing it.

Under dynamic conditions, ship stability is measured by the area under the statical stability curve. The latter is normally obtained from the cross curves of stability. Various methods are available for the calculation of these curves. These methods, however, are based on the assumption that inertia forces and hydrodynamic pressure are neglected. Therefore, experimental and theoretical methods are proposed to determine ship stability among waves.

Because of the random variation of the main parameters affecting the shape and area under the statical stability curve, the characteristics of the latter should be treated as random variables. Consequently, the reserve of dynamical stability should be associated with the risk of capsizing since the external forces acting on a ship among waves are random in nature.

No attempt is made here to establish or examine stability standards and criteria. Problems regarding damage stability, stability of ships among waves, the carriage of bulk cargo and the effects of forward motion are outside the scope of this paper. The stability of deeply submerged ships, of fishing vessels, of planing hulls and of coastal vessels are studied elsewhere.

## Initial stability

Initial stability of displacement vessels is defined by the distance between the ship's centre of gravity, G, and the metacentre,  $M_0$ , see Fig. 1. A floating ship has a stable equilibrium, at small angles of inclination, when:

$$GM_0 > 0$$

Since  $GM_0$  does not determine the ability of a ship to resist the moments of external forces, it cannot be solely used as a general measure of ship stability. Therefore, initial stability should be measured by the coefficient of stability, ie,  $\Delta \cdot GM_0$ ,  $\Delta$  being the ship displacement.

The inadequacy of  $GM_0$  as a measure of ship stability could be verified by examining the statical stability curves of two different ships. The first has a small  $GM_0$  and a large freeboard. The second ship has a large  $GM_0$  and a small freeboard. It is evident from Fig. 2 that the first ship is much more stable than the second, which has a larger  $GM_0$ .

Although  $GM_0$  alone cannot be taken as a general measure of ship stability, a minimum value is generally specified by regulatory bodies and classification societies. The limiting value of  $GM_0$  depends on several parameters, among them are ship's type, size, loading etc, and in general should satisfy the following condition:

$$(GM_0)_{\min} > (GM_L)_{\max} + GM_R$$

where  $GM_L$  = lost GM due to flooding.

$GM_R$  = residual GM after flooding.

The magnitude of  $GM_R$  should be adequate enough to retain a damaged ship stable.

The initial stability,  $GM_0$ , is generally given by, see Fig 1:

$$GM_0 = KM_0 - KG \quad (1)$$

where;  $KM_0 = KB_0 + B_0M_0$

$B_0M_0$  = initial metacentric radius.

A ship has adequate initial stability when:

$$KM_0 - KG > 0$$

$$\text{i.e. } KM_0/KG > 1.0$$

KG is a variable quantity given by:

$$KG = (\Delta_0 \cdot KG_0 + \sum_{i=1}^n w_i \cdot z_i) / \Delta \quad (2)$$

where,  $\Delta_0$  = light ship displacement.

$KG_0$  = height of  $G_0$  of the light ship from base line.

It could be determined by an inclining experiment.

$w_i$  = weight of item  $i$ .

$z_i$  = height of C. G. of  $w_i$  from base line.

$n$  = total number of weight items.

$\Delta$  = loaded ship displacement.

For certain types of ships, such as oil tankers, KG may be treated as a deterministic quantity for the particular loading condition. For other types of ships, KG is a variable quantity having a lower and an upper limit given by:

$$(KG)_L < KG < (KG)_U \quad (i)$$

where L and U stand for lower and upper respectively.

It should be noted here that the presence of partially filled tanks adds to the variability of KG because of the free surface effect.

On the other hand,  $KM_0$  depends mainly on ship form, sea condition and loading. Therefore,  $KM_0$  is not a deterministic quantity and could be treated as a random variable having a lower and an upper limit given by:

$$(KM_0)_L < KM_0 < (KM_0)_U \quad (ii)$$

In order to illustrate the effect of the variability of KG and  $KM_0$  on the magnitude of  $GM_0$ , the probability density functions, p.d.f., of both KG and  $KM_0$  should be determined. However, because of the lack of adequate statistical data to establish the mathematical model representing the random variation of KG and  $KM_0$ , a truncated normal p.d.f. could be assumed as follows:

$$p_X(x) = f_X(x)/H \quad (3)$$

$$\text{where, } f_X(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma_x} \cdot \exp \left[ -\frac{1}{2} \left( \frac{x - \bar{x}}{\sigma_x} \right)^2 \right] \quad (4)$$

$x$  = KG or  $KM_0$ .

$\bar{x}$  = mean value of KG or  $KM_0$ , i.e.  $\overline{KG}$  or  $\overline{KM_0}$

$\sigma_x$  = standard deviation of KG or  $KM_0$ .

H is determined from the following condition:

$$\int_{-\infty}^{\infty} f_X(x) \cdot dx = 1.0$$

Hence, for a truncated distribution, H is given by:

$$H = \int_{-\infty}^{x_U} f_X(x) \cdot dx - \int_{-\infty}^{x_L} f_X(x) \cdot dx$$

Since KG and  $KM_0$  are statistically independent random variables, the p.d.f. of  $GM_0$  is also a truncated normal distribution.

In order to compute the risk of losing  $GM_0$ , the effect of truncation is neglected so as to simplify the computations.

$$\text{Hence, } R = \int_{-\infty}^0 p(GM_0) \cdot dGM_0 \quad (5)$$

The parameters of the p.d.f. of  $GM_0$  are:

$$\sigma_{GM_0}^2 = \sigma_{KM_0}^2 + \sigma_{KG}^2 \quad (b)$$

$$\text{where, } \sigma_{KG} \cong \delta KG/3, \sigma_{KM_0} \cong \delta KM_0/3$$

$2\delta KG$  and  $2\delta KM_0$  are the ranges of possible variation of KG and  $KM_0$  respectively, and are given by:

$$2\delta KG = (KG)_U - (KG)_L$$

$$2\delta KM_0 = (KM_0)_U - (KM_0)_L$$

The risk of losing  $GM_0$  is given by the shaded area shown in Fig (3).

In order to simplify the calculation of R, the coefficients of variation of KG and  $KM_0$  are used as follows:

$$u = \sigma_{KM_0}/\overline{KM_0} \quad \text{and} \quad v = \sigma_{KG}/\overline{KG}$$

$$\text{Hence, } R = P(GM_0 < 0) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^w \exp \cdot \left( -\frac{t^2}{2} \right) \cdot dt \quad (c)$$

$$\text{where } t = (GM_0 - \overline{GM_0})/\sigma_{GM_0}$$

$$w = (F - 1)/\sqrt{F^2 \cdot u^2 + v^2}$$

$$F = \overline{KM_0}/\overline{KG} > 1.0$$

F represents a factor of safety against the loss of initial stability.

The effect on R of variation of u, v and F are given in Table 1. It can be seen that when  $u = 0.1$ ,  $v = 0.1$  and  $F = 1.2$ , the risk of losing initial stability is about 1/10.

It should be indicated here that losing initial stability does not necessarily lead to capsizing but may cause the ship to attain a list whose magnitude depends on the shape of the static stability curve. However, losing  $GM_0$  during launching or drydocking may lead to a disaster. In this case, careful attention should be given to the estimation of  $GM_0$ .

### Dynamical stability

Dynamical stability represents the work done,  $A_R$ , by the righting moment,  $M_R$ , in inclining a ship through an angle  $\theta$ , and is given by, see Fig 4:

$$A_R = \int_0^{\theta} M_R \cdot d\theta = \Delta \int_0^{\theta} GZ \cdot d\theta$$

where GZ = righting arm,  $\Delta$  = ship displacement.

Dynamical stability could be also considered as the work done by the weight of the ship over the change in the vertical distance between G and B, B being the centre of buoyancy. Therefore, the variation in the positions of B and G play the principal role in the calculation of dynamical stability.

Under the action of an arbitrary heeling moment,  $M_H$ , the equilibrium position is determined from the following condition:

$$A_R = A_H$$

$$\text{where, } A_R = \int_0^{\theta_d} M_R \cdot d\theta \quad \text{and} \quad A_H = \int_0^{\theta_d} M_H \cdot d\theta$$

$\theta_d$  = dynamic angle of heel, see Fig 5.

In order to ensure adequate dynamical stability, the following condition must be satisfied:

$$A_R > A_H$$

$$\text{or } S_D = A_R - A_H > 0$$

where  $S_D$  = reserve of dynamical stability, see Fig 6.

The limiting value of a heeling moment independent of the angle of heel  $\theta$  could be determined as shown in Fig 6. In this case, the reserve of dynamical stability vanishes.

The risk of capsizing, R, is calculated from the p.d.f. of the reserve of dynamical stability as follows:

$$R = P(S_D < 0) = \int_{-\infty}^0 p(S_D) \cdot dS_D$$

The p.d.f. of  $S_D$  is obtained from the p.d.f.s of  $A_R$  and  $A_H$ . The heeling moment,  $M_H$ , is a random variable since it depends largely on the wind and sea conditions. Consequently,  $A_H$  is also a random variable.

The variability of  $A_R$  results from the variabilities of:

- (1) initial stability,  $GM_0$ .
- (2) the magnitude of the maximum righting arm,  $GZ_m$ .
- (3) angle of vanishing stability,  $\theta_v$ .

The variability of  $GM_0$  has been discussed earlier. The effect of a small change in  $GM_0$  on the magnitude of  $GZ$  is given in the Appendix. The variability of  $GZ_m$  and  $\theta_v$  results from:

- (a) free surface effects and the possibility of shifting of cargo, fittings etc.
- (b) the effect of trim resulting from heeling the ship.
- (c) sea condition.
- (d) errors inherent in the calculation of the cross-curves of stability.
- (e) errors in the estimation of  $KG$ .

In order to determine the risk of capsizing,  $R$ , the p.d.f.s of both  $A_R$  and  $A_H$  should be determined. However, in the absence of adequate statistical data to establish the mathematical model representing the random variation of  $A_R$  and  $A_H$ , a truncated normal p.d.f. could be assumed. By virtue of statistical independence,  $S_D$  follows also a truncated normal p.d.f. A further simplification in the calculation of  $R$  could be achieved by assuming a normal p.d.f. for  $A_R$  and  $A_H$ . Consequently, the calculation of  $R$  follows the same procedure given before for calculating the risk of losing  $GM_0$ . Therefore, the risk of capsizing could be determined from Table 1 or Fig 7. In this case, the coefficients of variation  $u, v$  are given by:

$$u = \sigma_{A_R} / \bar{A}_R, \quad v = \sigma_{A_H} / \bar{A}_H$$

$$\text{and } F = \bar{A}_R / \bar{A}_H > 1.0, \quad w = \bar{S}_D / \sigma_{S_D}$$

$$\text{where, } \bar{S}_D = \bar{A}_R - \bar{A}_H$$

$$\sigma_{S_D}^2 = \sigma_{A_R}^2 + \sigma_{A_H}^2$$

$$\bar{A}_R = \text{mean value of } A_R \text{ and } \bar{A}_H = \text{mean value of } A_H$$

$$\sigma_{A_R} \cong \delta A_R / 3, \quad \sigma_{A_H} \cong \delta A_H / 3$$

$2\delta A_R$  and  $2\delta A_H$  are the ranges of possible variation in  $A_R$  and  $A_H$  respectively.

the risk of capsizing is approximately 1/4 and when  $u = 0.05$ ,  $v = 0.05$  and  $F = 1.1$ , the risk of capsizing is 1/11, see Fig 7.

From the foregoing analysis, it is evident that the risk of capsizing is greatly influenced by the shape of the statical stability curve. The latter is normally obtained from the cross curves of stability. Therefore, improving the accuracy of calculating these curves cannot be overemphasised. The effect on  $R$  of variation in  $KG$ , or  $GM_0$ , could be realised from the number of capsized ships carrying deck loads.

It should be mentioned here that the area under the statical stability curve does not give a correct measure of dynamical stability as it does not take into account such factors as inertia and hydrodynamic forces as well as sea condition and the trim associated with heeling. However, the risk of capsizing based on the area under the statical stability curve could be used for qualitative measures or for comparing different designs.

## Conclusions

- (1) Initial stability cannot be solely used as a general measure of the stability of displacement vessels.
- (2) Deficient initial stability not, only affects ship stability under static conditions, but it also has an adverse effect on dynamical stability.
- (3) Statistical data are required to establish the mathematical model representing the random variation of the relevant parameters associated with initial stability and the reserve of dynamical stability.
- (4) The risk of capsizing is greatly influenced by the magnitude of initial stability and the shape of the statical stability curve.
- (5) The risk of capsizing should be estimated and associated with the required minimum value of the reserve of dynamical stability.

## Appendix

The main characteristics of the statical stability curve are:

- (1) slope at the origin,  $\psi$ , see Fig 8, which is given by:
 
$$\tan \psi = \Delta \cdot GM_0$$
- (2) maximum righting moment,  $(M_R)_m$
- (3) angle of vanishing stability,  $\theta_v$
- (4) area under the righting moment curve,  $A_R$ , i.e.

u	v	R × 10 <sup>3</sup>				
		F				
		1.1	1.2	1.4	1.6	2.0
0.0	0.05	22.75	0.032	0.0	0.0	0.0
	0.10	158.6	22.75	0.032	0.0	0.0
	0.20	305.0	158.6	22.75	1.35	0.0
0.05	0.05	90.0	5.22	0.0016	0.0	0.0
	0.10	185.0	43.2	0.52	0.0013	0.0
	0.20	310.0	169.0	29.53	2.67	0.004
0.10	0.05	205.0	61.96	3.50	0.172	0.0
	0.10	250.0	100.2	10.04	0.736	0.004
	0.20	350.0	195.6	50.66	9.57	0.202
0.20	0.05	325.0	207.0	79.8	31.97	6.55
	0.10	340.0	220.8	89.25	36.75	7.64
	0.20	365.0	261.0	122.5	55.92	12.67

Table 1, the risk of losing  $GM_0$  or capsizing

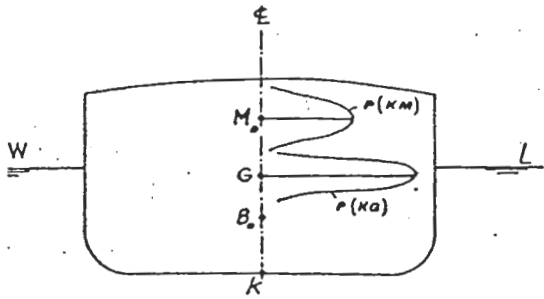


Fig 1. left

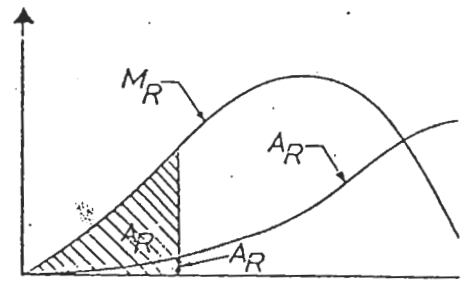


Fig 4. above

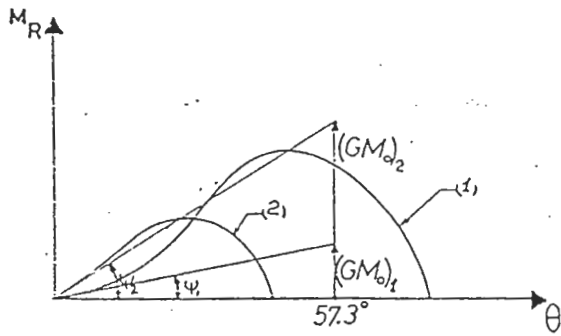
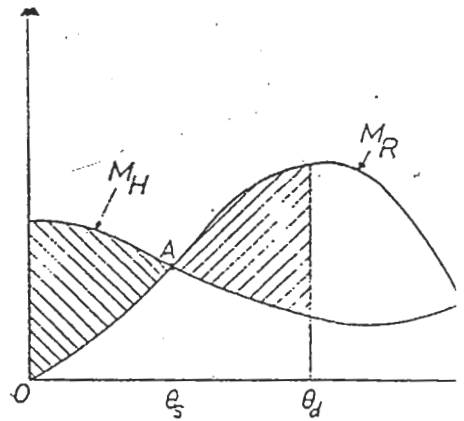
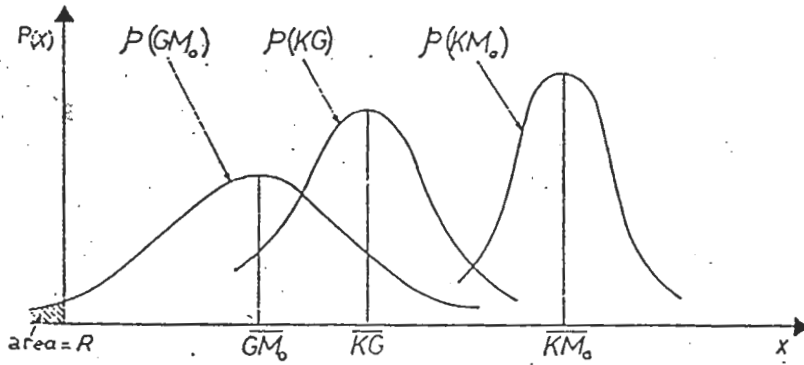


Fig 2. left

- 1- large freeboard
- 2- small freeboard

Fig 3. below



A=stable equilibrium  
B=unstable equilibrium

Fig 5. above

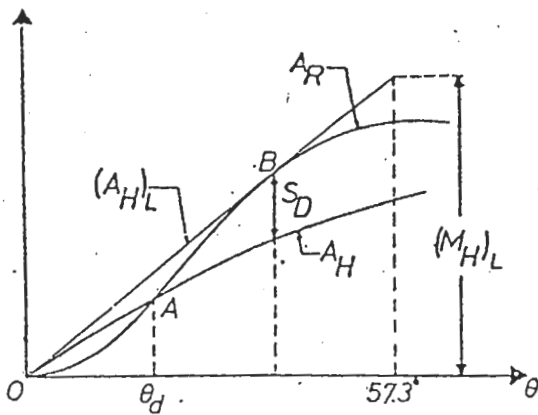


Fig 6. left

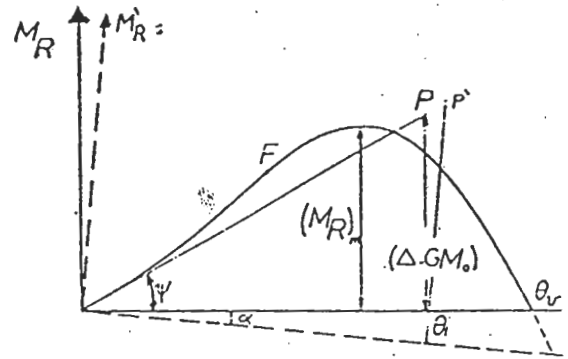


Fig 8. above

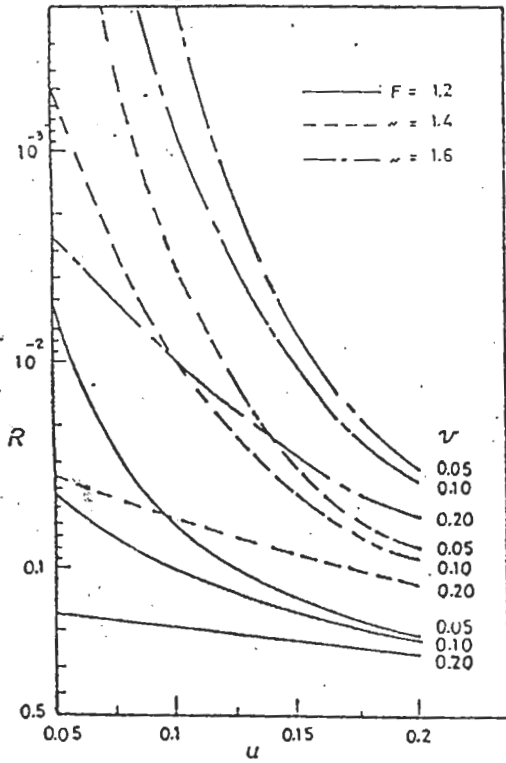


Fig 7. left

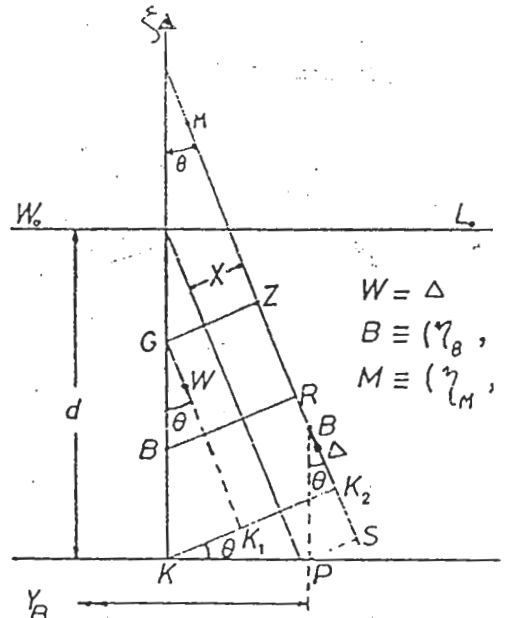


Fig 9. above